Phase transition in a computer network traffic model

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We propose and study here a simple model of computer network traffic that can exhibit a phase transition from a low to high congestion state measured in terms of average travel time of packets as a function of the packet creation rate in the network. In the model, packets are generated with destination addresses, and are transferred from one router to another toward their destinations. The routers are capable of queuing packets and autonomously selecting a path to the next router for a packet. Through simulations on a two-dimensional lattice model network, we found that the phase transition point into the congestion phase depends on how each router chooses a path for the packets in its queue. In particular, an appropriate randomness in path selection can shift the onset of traffic congestion to accommodate more packets in the model network. [S1063-651X(98)04107-5]

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I. INTRODUCTION

The phenomenology of the nature of computer network traffic has commanded much attention recently. Analysis, simulations, and experiments based on such concepts as ''phase transitions'' and ''self-similarity'' are active research topics in physics as well as in computer science (see, e.g., [1]). Against this background this paper also addresses the phase transition nature of computer network traffic. Our main focus, however, is not so much on understanding the nature of traffic itself, rather we concentrate on shifting of phase transition points from a low to high congestion state as we change path selection strategy for packets. Our approach is first to create a simple simulation model of network traffic that shows phase transition for the average travel time of packets as a function of packet generation rate in the network. We then propose a probabilistic routing strategy that can shift the phase transition point. We show that, with a suitable value for a parameter to control randomness in the path selection, onset of the phase transition into the congestion phase is ''eased,'' i.e., the network can accommodate a higher rate of packet generation before it goes into that phase. To gain more insight into this effect, we vary the number of routers that take this probabilistic strategy. We found that the effectiveness of the model shows a nonlinear response as a function of the proportion of probabilistic routers. From this result we infer that the shifting of the phase transition point is due to interactions between these adaptive routers rather than as a sum of separate contributions from individual units. We conclude with a discussion of this simulation model.

II. THE MODEL

The network architecture considered in the model consists of nodes placed as a two-dimensional lattice [Fig. 1(a)]. It is a square with N nodes (routers) on each side and N^2 nodes as a whole. Packets are generated and destroyed on nodes on the boundary of the lattice (squares in the figure), but not on inner nodes (circles). Inner nodes only forward packets received from neighbor nodes. A node on the boundary generates a packet according to the Poisson arrival with λ and sends it to a destination node selected randomly from among the nodes on the boundary (including itself). Each node has a receiving queue of unlimited length through which packets are forwarded to the destination and then destroyed.

During each unit time, each node goes through the following process in order to forward packets. It picks the packet at the head of its queue, decides to which node among its neighbors it will forward the packet, and then puts it at the end of the queue of the selected node. The next node is selected so that the packet is delivered to its destination along the shortest path. If more than one candidate of the next node exist, a strategy is needed to select the recipient. In our simulation we consider two strategies. One of them makes the decision deterministically, which we call ''deterministic routing,'' and the other does so probabilistically, which we call ''probabilistic routing.'' With deterministic routing, the node among the candidates to which the least number of packets has been forwarded so far is selected as the next node.

The probabilistic routing strategy that we compare with this deterministic routing is given by introducing a particular form of routing probability function. When we have multiple routes A and B based on the destination address, we assign the probability to choose a route A or B by the following equation:

$$P(A)=\frac{e^{-\beta X_A}}{e^{-\beta X_A}+e^{-\beta X_B}}, \tag{1}$$

$$P(B)=\frac{e^{-\beta X_B}}{e^{-\beta X_A}+e^{-\beta X_B}}, \tag{2}$$

$$1=P(A)+P(B), \tag{3}$$

where β is a parameter. X_A and X_B are the number of packets the router has already sent in the direction of A and B.

We note a couple of points about this probability function. First, the main difference with the deterministic routing is

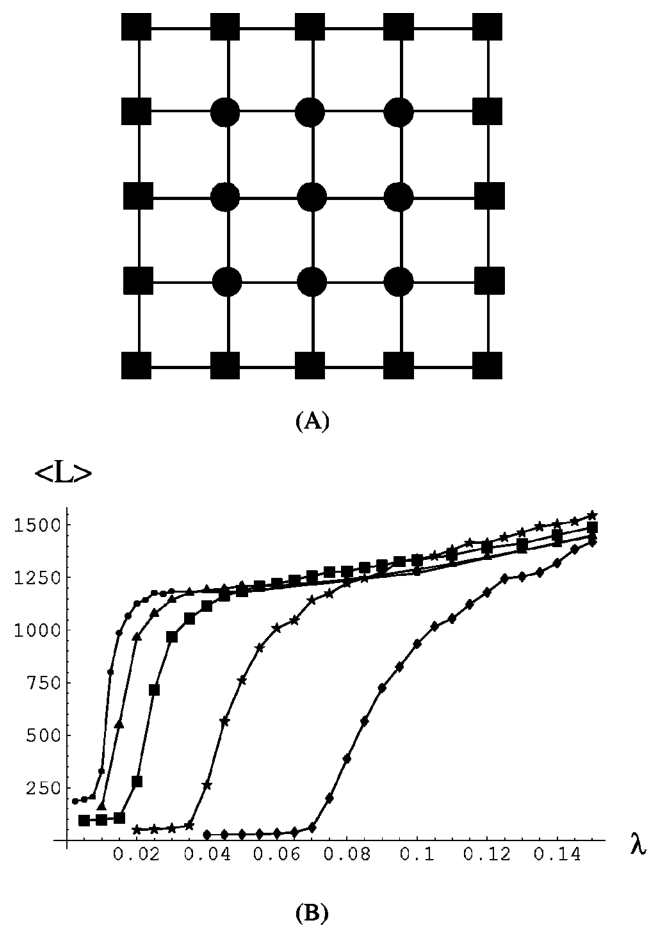

FIG. 1. Model network architecture and phase transition behavior with deterministic routing measured in average lifetime of a packet $\langle L\rangle$ as we vary packet creation rate λ. The system size is varied as $N=$ 25 (diamond), 50 (star), 100 (square), 150 (triangle), and 250 (circle). The lines are drawn for the reader's convenience.

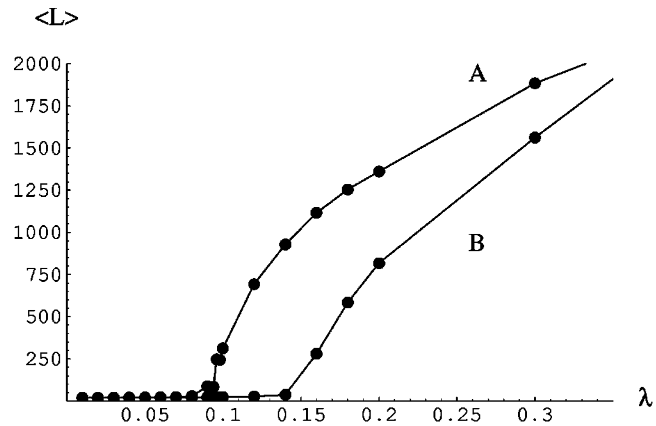

FIG. 2. Comparison of the phase transition behavior of the deterministic (a) and the probabilistic ($\beta=0.008$) (b) routing for $\langle L\rangle$. The system size is $N=25$. A similar graph is obtained for other values of N.

that even if $X_A > X_B$, there is some probability of choosing and sending a packet to A. Second, we can recover the deterministic model by letting $\beta\to\infty$. Also, if we set $\beta=0$, we have a completely random choice of A and B, i.e., A and B will be chosen with equal probability of 0.5 regardless of X_A and X_B. Hence, β is a control parameter that determines the degree of randomness of path selection.

III. SIMULATION RESULTS

We quantify network traffic congestion by the average ''lifetime'' of a packet $\langle L\rangle$, which is the average time between the sending and receiving of a packet. (Averages are taken over packets.) In Fig. 1(b) we show the behavior of $\langle L\rangle$ as we change the creation λ of the packet using the deterministic routing. The simulation is performed with various system sizes of N, and the system is run up to time step 10 000. The phase transition behavior is clearly observable as λ increases beyond a ''critical rate,'' λ_c. This transition into the congestion phase is sharper with increasing size, as seen in other physical systems showing phase transitions. Such sudden change into a congestion state with increasing flow of packets is observed in a real computer network. Thus, even though our model is simple, omitting several properties of real computer networks, it can capture some qualitative behavior of the traffic.

We now turn our attention to comparison of the deterministic routing with the proposed probabilistic routing. One such example is shown in Fig. 2. We can see that the onset of phase transition is moved to higher λ, showing that the model network with probabilistic routing can tolerate more packets before going into a congestion phase.

To examine the effect of randomness for this shift of phase transition point, we plot in Fig. 3(a) the phase transition points as a function of β. We see that we need to choose β appropriately [≈ 0.01 in Fig. 3(a)] to have the optimal phase point shift. This existence of an optimal amount of randomness for system performance is similar to the cases of ''stochastic resonance'' [3] and ''simulated annealing'' [4]. For individual routers, the deterministic routing appears to be the most balanced way of sending packets to the next router. Use of the probabilistic routing strategy means that this balance is sometimes intentionally upset. The fact that easing of the phase transition point nonetheless takes place means that an emergent collective behavior of routers is playing a crucial role in deciding the congestion nature of the network.

To gain more insight into the collective behavior of the model, we investigate how the phase transition point changes when only a portion of the routers have the probabilistic routing and others operate using the deterministic routing; a representative example is shown in Fig. 3(b). We see that the critical points of phase transition change nonlinearly and show saturation as a function of the proportion of probabilistic routers. From a system design point of view, this response shape indicates a possibility of fault tolerance: the effectiveness of the system does not deteriorate appreciably until a certain proportion of routers become nonprobabilistic. This nonlinear shape together with phase transition behavior suggests that the collective behavior of the model is not simply an aggregation of the effect of individual routers. Rather, the interaction among routers, which is indirectly mediated by packets passing through, is playing a role in the collective behavior of the model system.

IV. DISCUSSION

The emergent behavior observed here with our model is rather intricate to analyze theoretically, particularly in deal-

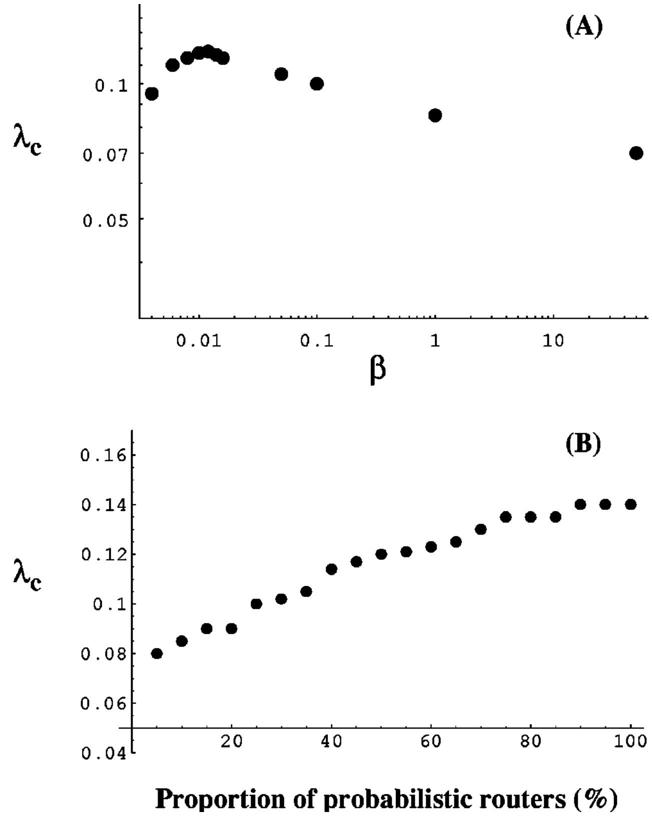

FIG. 3. (a) Change of "critical rate," λ_c as β is varied for probabilistic routing. The system size is $N=25$. (b) Change of critical rate, λ_c, as the proportion of the probabilistic routers is varied in the system. The system size is $N=20$ and $\beta=0.008$.

ing with packet destination information. Hence, we are at this point relying on computer simulations to find a shift of phase transition points and effective β. We can, however, qualitatively view the model behavior as a result of interaction between packet flow and routers collectively: packet flow affects the behavior of routers, which in return regulates the flow. We are currently studying a dynamic traffic pattern formation for this two-dimensional model as well as searching for an analytical framework in which to study it.

As a related topic, the traffic flow problem on motorways has also been investigated through theoretical modelings. There are models using fluid dynamics analogy, cellular automata, and so on [2]. There is also a series of research reports on controlling automobile traffic [5]. We have recently proposed an autonomous distributed signal control model in that context [6]. We report here briefly that average velocity of the model traffic with signals has shown improvement with an autonomous adaptive dynamics strategy for signal periods. There, as in this network traffic model, a collective behavior is important, and a self-organized periodic pattern of traffic emerged.

It should again be noted that the probabilistic routing scheme proposed here does not explicitly contain a procedure to adjust for better performance, and individual routers occasionally send packets to a direction that appears to increase congestion. Hence, the effectiveness of the routing scheme only emerges as the collective behavior of the entire system. In this sense these models belong to a class of models often termed emergent computation models [7,8]. These models aim to derive effective computation from the interaction of autonomous local actions from each unit in the system rather than by top-down style algorithms or by hierarchical controls with or without feedbacks, which are commonly seen with more realistic network traffic control schemes. (See, e.g., [9,10].) Even though the guiding principles of designing such emergent computation models vary, experience, concepts, and insight gained from studies of systems showing emergent behaviors such as in neural network modelings [11] can be a useful guide for theoretical understanding of this routing model and its behavior, and remains to be undertaken in the future.

[1] E. W. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson, ACM/SIGCOMM Comput. Commun. Rev. **23**, 183 (1993); S. Yukawa and M. Kikuchi, J. Phys. Soc. Jpn. **64**, 35 (1995); M. Takayasu, H. Takayasu, and T. Sato, Physica A **233**, 824 (1996); M. S. Taqqu, W. Willinger, and R. Sherman, ACM/SIGCOMM Comput. Commun. Rev. **27**, 5 (1997); A. U. Tretyakov, H. Takayasu, and M. Takayasu, Physica A **233**, 315 (1998).

[2] W. Leutzbach, *Introduction to the Theory of Traffic Flow* (Springer-Verlag, Berlin, 1988); O. Biham, A. A. Middleton, and D. Levine, Phys. Rev. A **46**, R6124 (1992); M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, Phys. Rev. E **51**, 1035 (1995).

[3] K. Wiesenfeld and F. Moss, Nature (London) **373**, 33 (1995); A. R. Bulsara and L. Gammaitoni, Phys. Today **49** (3), 39 (1996).

[4] S. Kirkpatric, C. Gelatt, and M. Vecchi, Science **220**, 671 (1983).

[5] K. W. Huddart, *Eighth International Conference on Road Traffic Monitoring and Control*, IEE Conf. No. 422 (IEE, London, 1996).

[6] T. Ohira, K. Inoue, and Y. Takeshima, in *Progress in Connectionist-based Information Systems*, edited by N. Kasabov *et al.* (Springer, Singapore, 1997), p. 939.

[7] S. Forrest, *Emergent Computation* (MIT Press, Cambridge, 1991); B. A. Huberman, *Computation: The Micro and The Macro View* (World Scientific, Singapore, 1992).

[8] At the initial submission of this paper, we were pointed to the following works, which present other network traffic control models with this type of approach: I. Campos, A. Tarancón, F. Clérot, and L. A. Fernández, Phys. Rev. E **52**, 5946 (1995); S. Shenker, Comput. Commun. Rev. **24** (3), 47 (1994).

[9] A. Charny, D.D. Clark, and R. Jain, in *Proceedings of the International Conference on Communications* (IEEE, Los Alamitos, CA, 1995), p. 1954.

[10] W. Stallings, *High-Speed Networks: TCP/IP and ATM Design Principles* (Prentice-Hall, London, 1998).

[11] L. Arbib, *The Handbook of Brain Theory and Neural Networks* (MIT Press, Cambridge, 1995).